Number System

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Major topics:

1. Binary operations and
2. Base Conversions

Chapter Outline

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division
- Decimal to Binary conversion
- Decimal to Octal conversion
- Decimal to Hexadecimal conversion
- Binary to Decimal
- Binary to Octal
- Binary to Hexadecimal
- Octal to Decimal
- Octal to Binary
- Octal to Hexadecimal
- Hexadecimal to Decimal
- Hexadecimal to Binary
- Hexadecimal to Octal

Information Representation

- Binary Representation in Computer System
  - ❖All information of diverse type is represented within computers in the form of ***bit patterns/ binary string…..10000110000***.
    - Examples: text, numerical data, sound, images, videos etc
  - ❖These information is not in human readable forms, that's why we need I/O interfaces to represent it in human understandable form.
  - ❖One important aspect of computer design is to decide how information is converted ultimately to a bit pattern
  - ❖Hence, to understand the computer architecture it is essential to have a basic knowledge of number system.

Number System basic types

Non-Positional Number System

- Uses an additive approach for counting
- For example I for 1, II for 2, IIII for 4, IIIII for 5 etc.
- Each symbol represents same value regardless of its position
- Used in early days for counting methods

Positional Number System

- Uses few finite symbols known as **Digits**.
- Digit represents different values and is determined by below three considerations:

1. The digit itself
2. The position of the digit in the number.
3. The base of the number system

Different number system on the basis of their base values

- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

Decimal Number System

- Base is 10 or ‘D’ or ‘Dec’
- Uses in our daily life
- Use Ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Each place is weighted by the power of 10

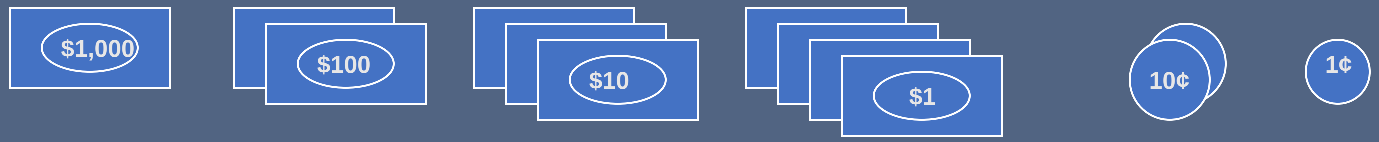
Example:

1234_{10} *or* 1234_D

$= 1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$= 1000 + 200 + 30 + 4$

[note: $x^0 = 1$, hence $2^0 = 1$, $8^0 = 1$, $10^0 = 1$, $16^0 = 1$]



Binary Number System

- Base is 2 or ‘B’
- Use in discrete system like digital computer
- Use Two symbols: 0 and 1
- Each place is weighted by the power of 2

Example:

1111_2 *or* 1111_B

$= 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 2 + 1$

[note: $x^0 = 1$, hence $2^0 = 1$, $8^0 = 1$, $10^0 = 1$, $16^0 = 1$]

8 4 2 1

Octal Number System

- Base is 8 or ‘O’ or ‘Oct’
- Use to represent binary string in short form and each digit represents 3 binary bits. For example: 7 represents 111
- Use Eight symbols: 0,1,2,3,4,5,6 and 7
- Each place is weighted by the power of 8

Example:

5237_8 *or* 5237_O or 5237_{Oct}

$= 5 \times 8^3 + 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$

$= 5 \times 512 + 2 \times 64 + 3 \times 8 + 7 \times 1$

[note: $x^0 = 1$, hence $2^0 = 1$, $8^0 = 1$, $10^0 = 1$, $16^0 = 1$]

512	64	8	1

Hexadecimal Number System

- Base is 16 or ‘H’ or ‘Hex’
- Use to represent binary string in short form and each digit represents 4 binary bits. For example: 9 represents 1001
- Use Sixteen symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A(=10), B(=11), C(=12), D(=13), E(=14), F(=15)

- Each place is weighted by the power of 16

Example:

$A941_8$ *or* $A941_O$ or $A941_{Oct}$

$= A \times 16^3 + 9 \times 16^2 + 4 \times 16^1 + 1 \times 16^0$

$= 10 \times 4096 + 9 \times 256 + 4 \times 16 + 1 \times 1$

[note: $x^0 = 1$, hence $2^0 = 1$, $8^0 = 1$, $10^0 = 1$, $16^0 = 1$]

4096	256	16	1

Number System summary table

Number System	Base value	Set of symbols	Example
Decimal	10	(0,1,2,3,4,5,6,7,8,9)	$(29)_{10}$
Binary	2	(0,1)	$(1011100101)_2$
Octal	8	(0,1,2,3,4,5,6,7)	$(1752)_8$
Hexadecimal	16	(0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)	$(\text{B6F4})_{16}$

Things to remember: Each number system is just a different way for representing the quantities. Moreover, the quantities remains the same, but the symbols used to represent those quantities are changed in each number system.

Number System equivalence table

Binary	Octal	Decimal	Hexadecimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	10	A
1011	13	11	B
1100	14	12	C
1101	15	13	D
1110	16	14	E
1111	17	15	F

1.Decimal to Binary Number System $(\ )_{10}$ to $(\ )_2$

Rules:

1. Divide the decimal number by base value of binary(2) and write the remainder.
2. Repeat the rule 1 till the quotient becomes zero.
3. Write the remainders from bottom to top.

Example: $(19)_{10} = (?\ )_2$

Divide-by-2	Quotient	Remainder
19/2	9	1
9/2	4	1
4/2	2	0
2/2	1	0
1/2	0	1

Stop, since quotient = 0.
Hence, $(19)_{10} = (10011)_2$
Question 1: $(23)_{10} = (?)_2$

2.Decimal to Octal Number System $(\ )_{10}$ to $(\ )_8$

Rules:

1. Divide the decimal number by base value of Octal(8) and write the remainder.
2. Repeat the rule 1 till the quotient becomes zero.
3. Write the remainders from bottom to top.

Example: $(109)_{10} = (?\ )_8$

Divide-by-2	Quotient	Remainder
1099/8	137	3
137/8	17	1
17/8	2	1
2/8	0	2

Stop, since quotient = 0.
Hence, $(1099)_{10} = (2113)_8$
Question 2: $(203)_{10} = (?)_8$

3.Decimal to Hexadecimal Number System $(\ )_{10}$ to $(\ )_{16}$

Rules:

1. Divide the decimal number by base value of Hexadecimal(16) and write the remainder.
2. Repeat the rule 1 till the quotient becomes zero.
3. Write the remainders from bottom to top.

Example: $(8029)_{10} = (?\ )_{16}$

Divide-by-2	Quotient	Remainder	Hexadecimal Digit
8029/16	501	13	13=D
501/16	31	5	5=5
31/16	1	15	15=F
1/16	0	1	1=1

Stop, since quotient = 0.

Hence, $(8029)_{10} = (1F5D)_{16}$

Question 3: $(203)_{10} = (?)_{16}$

4. Binary to decimal Number System $(\ )_2$ to $(\ )_{10}$

Rules:

1. Multiply each binary digits with its place value i.e. powers of two with its positional weight.

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

2. Sum all the products.

Binary to Decimal generalized formula:

$$(a_{n-1}\, a_{n-2} \ldots\ldots a_1\, a_0)_{\mathbf{2}} = (a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots + a_1 \times 2^1 + a_0 \times 2^0)_{\mathbf{10}}$$

<u>Example</u>: $(101011)_2 = (1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0)_{10}$

$= (1 \times 32 + 1 \times 8 + 1 \times 2 + 1 \times 1)_{10}$

$= (43)_{10}$

Question 4: $(10111)_2 = (?)_{10}$

5.Octal to decimal Number System $( )_8$ to $( )_{10}$

Rules:

1. Multiply each octal digits with its place value i.e. powers of eight with its positional weight.

   $8^0 = 1$

   $8^1 = 8$

   $8^2 = 64$

   $8^3 = 512$

2. Sum all the products.

Binary to Decimal generalized formula:

$$(a_{n-1}\, a_{n-2} \ldots\ldots a_1\, a_0)_8 = (a_{n-1} \times 8^{n-1} + a_{n-2} \times 8^{n-2} + \ldots + a_1 \times 8^1 + a_0 \times 8^0)_{10}$$

<u>Example</u>: $(7351)_8 = (7 \times 8^3 + 3 \times 8^2 + 5 \times 8^1 + 1 \times 8^0)_{10}$

$= (7 \times 512 + 3 \times 64 + 5 \times 8 + 1 \times 1)_{10}$

$= (3584 + 192 + 40 + 1)_{10}$

$= (3817)_{10}$

Question 5: $(562)_8 = (?)_{10}$

6.Hexadecimal to decimal Number System $(\ )_{16}$ to $(\ )_{10}$

Rules:

1. Multiply each hexadecimal digits with its place value i.e. powers of sixteen with its positional weight.

   $16^0 = 1$

   $16^1 = 16$

   $16^2 = 256$

   $16^3 = 4096$

1. Sum all the products.

Binary to Decimal generalized formula:

$(a_{n-1}\, a_{n-2} \ldots\ldots a_1\, a_0)_{\mathbf{16}} = (a_{n-1} \times 16^{n-1} + a_{n-2} \times 16^{n-2} + \ldots + a_1 \times 16^1 + a_0 \times 16^0)_{\mathbf{10}}$

<u>Example</u>: $(F42B)_{16} = (F \times 16^3 + 4 \times 16^2 + 2 \times 16^1 + 11 \times 16^0)_{10}$

$= (15 \times 4096 + 4 \times 256 + 2 \times 16 + 11 \times 1)_{10}$

$= (43)_{10}$

Question 6: $(C2A)_{16} = (?)_{10}$

7.Binary to Octal Number System $(\ )_2$ to $(\ )_8$

Rules:

1. Group binary numbers in 3 bits group from right to left. (why 3 bits ?)
2. While grouping, if any digits are not enough for such group of 3 bits, then add zeros before the number.

    4 – 2 – 1

    100 – 010 – 001

3. Write its equivalent value of octal.

For example: $(10110101)_2 = (?)_8$

Binary number	10110101		
3-bits group	010	110	101
Equivalent Octal	2	6	5

Hence, $(10110101)_2 = (265)_8$

Question 7: $(10111011)_2 = (?)_8$

8.Binary to Hexadecimal Number System $(\ )_2$ to $(\ )_{16}$

Rules:

1. Group binary numbers in 4 bits group from right to left. (why 4 bits ?)
2. While grouping, if any digits are not enough for such group of 4 bits, then add zeros before the number.

8	–	4	–	2	–	1
1000	–	0100	–	0010	–	0001

3. Write its equivalent value of octal.

For example: $(10110101110)_2 = (?)_{16}$

Binary number	10110101110		
3-bits group	0101	1010	1110
Decimal Equivalent	5	10	14
Hexadecimal Equivalent	5	A	E

Hence, $(10110101110)_2 = (5AE)_{16}$

Question 8: $(10010111101)_2 = (?)_{16}$

9.Octal to Binary Number System $( )_8$ to $( )_2$

Rules:

1. Write the 3 bits combination of binary number for each octal digit.
2. While converting to 3 bits for each digit, if any digit has less than 3 bits, then add zero bit before it.

   0100 – 0010 – 0001

   4 – 2 – 1

3. Now write all the binary numbers from left to right.

For example: $(731)_8 = (?)_2$

Octal number	7	3	1
3 bits Binary Equivalent	111	011	001
Binary Equivalent	111011001		

Hence, $(731)_8 = (111011001)_2$

Question 9: $(6420)_8 = (?)_2$

10.Hexadecimal to Binary Number System $( )_{16}$ to $( )_2$

Rules:

1. Write decimal equivalent of each hexadecimal number.

2. Write the 4 bits combination of binary number for each decimal digit.

1. While converting to 4 bits for each digit, if any digit has less than 4 bits, then add zero bit before it.

1000	–	0100	–	0010	–	0001
8	–	4	–	2	–	1

3. Now write all the binary numbers from left to right.

For example: $(D2E)_{16} = (?)_2$

Hexadecimal number	**D**	**2**	**E**
Decimal equivalent	13	2	14
4 bits Binary group	1101	0010	1110
Binary Equivalent	110100101110		

Hence, $(D2E)_{16} = (110100101110)_2$

Question 10: $(BC51)_{16} = (?)_2$

11. Octal to Hexadecimal Number System $( )_8$ to $( )_{16}$

Rules:

1. Write the 3 bits combination of binary number for each octal digit.
2. Now form 4 bits combination of binary number from the right hand side.
3. Write the equivalent decimal value for each 4 bits group.
4. Write the equivalent Hexadecimal value for each 4 bits group

For example: $(573)_8 = (?)_{16}$

Octal number	**5**	**7**	**3**
3 bits binary group	101	111	011
4 bits Binary group	0001	0111	1011
Decimal equivalent	1	7	11
Hexadecimal equivalent	1	7	B

Hence, $(573)_8 = (17B)_{16}$

Question 11: $(634)_8 = (?)_{16}$

Alternative method: First convert octal into decimal number. Then convert decimal number to Hexadecimal number

12. Hexadecimal to Octal Number System $( )_{16}$ to $( )_8$

Rules:

1. Write the equivalent decimal value for each Hexadecimal digit.
1. Write the 4 bits combination of binary number for each decimal digit.
2. Now form 3 bits combination of binary number from the right hand side.
4. Write the equivalent octal value for each 3 bits group

For example: $(A2C)_{16} = (?)_8$

Hexadecimal number	**A**		**2**	**C**
Decimal equivalent	10		2	12
4 bits Binary group	1010		0010	1100
3 bits binary group	101	000	101	100
Octal equivalent	5	0	5	4

Hence, $(A2C)_{16} = (5054)_8$

Question 12: $(B4A)_{16} = (?)_8$

Alternative method: First convert Hexadecimal into decimal number. Then convert decimal number to octal number.

Computer uses a binary systems

Why binary?

1. Electronic bi-stable environment
   - on/off, high/low voltage
   - Bit: each bit can be either 0 or 1

2. Reliability
   - With only 2 values, can be widely separated, therefore clearly differentiated
   - “drift” causes less error

Binary operations

1. Addition

a	b	a + b
0	0	0
0	1	1
1	0	1
1	1	1 0

Carry bit

Can you remember this ?

1 + 1 + 1 = 11

1 + 1 + 1+ 1 =100

1+1+1+1+1=101

Practice questions:

a. 10111 + 11101
b. 1010 + 111

Example: $(111011)_2 + (10101)_2 = (?)_2$

1	1	1	1	1	← carry	
1	1	1	0	1	1	
+	1	0	1	0	1	
10	1	0	0	0	0	
Hence, $(111011)_2 + (10101)_2 = (1010000)_2$						

Binary Operations

2. Multiplication

a	b	a x b
1	1	1
1	0	0
0	1	0
0	0	0

Practice questions:

a. 1011 x 1110

b. 101 x 111

Example: $(1011)_2$ x $(111)_2$ = $(?)_2$

multiplicand		**1**	**0**	**1**	**1**
multiplier		x	1	1	1
		1	0	1	1
	1	0	1	1	x
1	0	1	1	x	x
10	1	1	1	0	1
Hence, $(1011)_2$ x $(111)_2$ = $(1011101)_2$					

Binary Operations

3. Subtraction

a	b	a - b
1	1	0
1	0	1
0	1	1 (borrows 1)
0	0	0

Remember this:

10 - 1 = 1

100 – 1 = 11

101 – 1= 100

Practice questions:

a. 10110 - 1011
b. 101 - 11

Example: $(11011)_2$ - $(1110)_2$ = $(?)_2$

minuend	1	1	0	1	1
subtrahend	-	1	1	1	0
Difference		1	1	0	1
Hence, $(11011)_2$ - $(1110)_2$ = $(1101)_2$					

Binary Operations

4. Division

a	b	a ÷ b
1	1	1
1	0	Not defined
0	1	0
0	0	0

Practice questions:

a. 10110 ÷ 101
b. 1010 ÷ 11

Example: $(11011)_2 \div (1110)_2 = (?)_2$

Divisor	Dividend	Quotient
1110	11011	1
	- 1110	
Remainder	1101	
Hence, $(1111)_2 \div (1110)_2 = (?)_2$ Quotient=1 and Remainder=1101		

Practice Exercises

Do question number 5 (a-l) and 7 (a-n) from the text book " Hamro computer science, Book-10" Author: Govinda Prasad Joshi